

Can a few fanatics influence the opinion of a large segment of a society?

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Received 31 August 2006 / Received in final form 18 December 2006

Published online 13 April 2007 – © EDP Sciences, Società Italiana di Fisica, Springer-Verlag 2007

Abstract. Models that provide insight into how extreme positions regarding any social phenomenon may spread in a society or at the global scale are of great current interest. A realistic model must account for the fact that globalization, internet, and other means of mass communications have given rise to scale-free networks of interactions between people. We propose a novel model which takes into account the nature of the interactions network, and provides some key insights into this phenomenon. These include, (1) the existence of a fundamental difference between a hierarchical network whereby people are influenced by those that are higher in the hierarchy but not by those below them, and a symmetrical network where person-on-person influence works mutually, and (2) that a few “fanatics” can influence a large fraction of the population either temporarily (in the hierarchical networks) or permanently (in symmetrical networks). Even if the “fanatics” disappear, the population may still remain susceptible to the positions originally advocated by them. The model is, however, general and applicable to any phenomenon for which there is a degree of enthusiasm or susceptibility to in the population.

PACS. 02.50.Ey Stochastic processes – 05.40.-a Fluctuation phenomena, random processes, noise, and Brownian motion – 89.65.-s Social and economic systems – 89.75.-k Complex systems

1 Introduction

Given the current political climate around the world, and the rise of extreme ideologies in many parts of the globe, models that can provide insight into how extreme ideologies and opinions spread in a society are clearly of current interest. In particular, one should keep in mind two facts: (1) globalization, the internet, and other modern means of long-distance communications (for example, fax, and mobile phones) have given rise to a scale-free (SF) network of interactions between people [1]. In a SF network the probability distribution $f(k)$ for a node to have k links to other nodes follows a power law,

$$f(k) \sim k^{-\gamma}, \quad (1)$$

where γ is a parameter that measures how well-connected the network is. Many unusual properties of SF networks have been attributed to the above distribution. (2) One should also keep in mind that although extreme ideologies are typically advocated by very small fringe groups, they may continue to survive and even thrive over certain time scales, it is clearly important to understand the role of such factors on the opinion of a population, and how they affect such antisocial behavior as terrorism.

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In this paper we propose a novel model in order to understand this phenomenon. The model is used to study how the opinions of various segments of a population may be influenced by the interactions among individuals, and how the connectivity of the interaction network influences the survival or disappearance of an opinion. We represent the network of interactions between people by a SF network [1] and study various scenarios that may affect the dynamics of the spreading of an opinion in a population.

The phenomenon that we study, and the model that we develop for it belong, in principle, to a general class of problems that describe various epidemic processes. In particular, our model and work are motivated by the work of Castillo-Chavez and Song [2] (see below). Great efforts have been devoted for decades to understanding how certain epidemic diseases, such as HIV, spread throughout a society [3,4]. In particular, the so-called SIS (susceptible-infected-susceptible), SIR (susceptible-infected-removed), and SEIR (susceptible-exposed-infected-recovered) models have been developed and studied either in terms of linear differential equations that describe the rate of change of each group of the population, or in discrete forms on regular lattices, such as the square lattice. The long-term dynamics of these models, when studied in terms of differential equations (which represent a type of mean-field approximation) or on regular lattices, is relatively simple [5] and can be expressed in terms of two fixed points: Either the disease dies out, or a stable equilibrium is reached

whereby the disease is endemic. A threshold condition determines which of the two fixed points is stable. More complex behavior may arise when, for example, the model contains a seasonal forcing. Generalizations to models in which the ill individuals have a continuum of states have also been made [6].

More recently, a few of such models have been examined in complex networks in order to understand some *social* phenomena. In particular, Zanette [7] examined the dynamics of an epidemiclike model for the spread of a “rumor” on a small-world (SW) network. A SW network is constructed starting from a one-dimensional lattice with periodic boundary conditions which, in effect, make the lattice a ring, where each node is connected to its $2k$ nearest neighbors, i.e., to the k nearest neighbors clockwise and counterclockwise [8]. To introduce disorder into the network, each of the k clockwise connections of each node i is rewired with a probability q to a randomly-selected node j that does not belong to the “neighborhood” of i . In this way, the lattice contains shortcuts between distant nodes. Zanette [7] showed that his model exhibits a transition between regimes of localization and propagation at a *finite* value of the network randomness q . Somewhat similar work was carried out by Shao et al. [9] who studied how “blackmail” propagates in a SW network. In contrast, Pastor-Satorras and Vespignani [10] showed that a dynamical model of spreading of epidemics does *not* exhibit any threshold behavior when studied in a SF network, hence demonstrating a crucial difference between spreading of an epidemic phenomenon in SW and SF networks, which is clearly due to their completely different connectivity structures.

The plan of this paper is as follows. In the next section we describe the model. Section 3 contains the results and a discussion of their implications. A modified shorter version was published for the social sciences in Sozialwissenschaftlicher Fachinformationsdienst(soFid) - Methoden und Instrumente der Sozialwissenschaften, 2006/1, Informations-Zentrum Sozialwissenschaften, Bonn, Germany.

2 The model

In the model the entire population is divided into four fractions: the general population G , those portions of the population that are either susceptible to, or excited about, an opinion, which we denote, respectively, by S and E , and the “fanatics” F who always advocate an opinion. Initially, everyone belongs to G , except a core of fanatics which, unless otherwise specified, is assumed to be four (but can be generalized to any number), since the most interesting results are obtained with a few initial fanatics (see below). Then, people can change their opinions depending on the neighbours to whom they listen to. Members of the S , E , and F groups can convince people in the G group to change their opinion and become susceptible to the fanatics’s opinion; members of the E and F groups can convince the S group to become E ; members of the F group can convince the E ’s members to convert

to F , but members of the S , E , and F groups can also directly return to the general population G . The fanatics are created initially by some outside event which is not part of the model. All the opinion changes happen with a probability p that can take any particular value if there is any evidence for it. Such a model can be applied not only to terrorism and other extreme opinions, but also to any other social phenomenon for which there is a degree of enthusiasm, or susceptibility, in a society.

A model of opinion dynamics was proposed recently based on the percolation model [11]. Another recent model [12] uses, similar to our work, SF networks, but its dynamics and the quantities that it studies are completely different from those of the model studied in this paper. The partition of the population and the probabilities of opinion change in our model are similar to the model of Castillo-Chavez and Song [2] who proposed a deterministic continuum model in terms of a set of nonlinear differential equations, given by

$$\begin{aligned}\frac{dS(t)}{dt} &= \beta_1 CG - \frac{\beta_2 S(E+F)}{C} - \gamma_1 S, \\ \frac{dE(t)}{dt} &= \frac{\beta_2 S(E+F)}{C} - \frac{\beta_3 EF}{C} - \gamma_2 E, \\ \frac{dF(t)}{dt} &= \frac{\beta_3 EF}{C} - \gamma_3 F,\end{aligned}\quad (2)$$

where the various coefficients, β_i and γ_i , are constant, and $C = S + E + F = 1 - G$. Without loss of generality, one can set $\beta_1 = 1$ since, otherwise, it can be absorbed in the time scale. (Omitting the denominators here does not change much the results.) For comparison, the dynamics of the SEIR model is described by [5]

$$\begin{aligned}\frac{dS(t)}{dt} &= \mu G - (\mu + \lambda)S, \\ \frac{dE(t)}{dt} &= \lambda S - (\mu + \sigma)E, \\ \frac{dF(t)}{dt} &= \sigma E - (\mu + \nu)F,\end{aligned}\quad (3)$$

and, $\lambda = \beta F$, with the various parameters being constant. It is clear that the dynamics of our model is, in the continuum limit, much more complex than that of the SEIR model. Castillo-Chavez and Song [2] studied their continuum model in detail. Similarly, the SEIR model was studied by, for example, Lloyd and May [5].

The models expressed by the sets (2) and (3) compute average behavior over the entire population and do not deal with individuals. Such approximations cannot answer, for example, the question of whether or how a few fanatics can convince an entire population about a certain opinion or proposition. They also cannot take into account the effect of the SF structure of the interaction network between people. Discretizing the model using a regular lattice, such as a square lattice, is also not realistic because the range of the interactions in such networks is limited. Instead, networks [1] between people or computers are described better as scale-free, and a network of the Barabási-Albert (BA) type is the most widespread.

$G \rightarrow S$ with probability β_1 , if the neighbour is S , E , or F ,
 $F \rightarrow G$ with probability γ_3 ,
 $E \rightarrow G$ with probability γ_2 , and $E \rightarrow F$ with probability β_3/C , if the selected neighbour is F ,
 $S \rightarrow G$ with probability γ_1 , and $S \rightarrow E$ with probability β_2/C , if the selected neighbour is E or F .

This is a complex network in which the probability distribution for a node to have k links to other nodes follows equation (1) with $\gamma = 3$. In such a network, a few people (nodes) have many connections, most people have rather few, and there is no sharp boundary between these extremes. We note that power laws also hold for the probability of terror attacks [13]. We simulate and study the model that we described above in the BA network which, to our knowledge, has never been done before on either the SW or SF networks. The BA networks are built by starting with four nodes (people) all connected to each other. Then newcomers join the network one after the other by connecting to the already-existing four members, with a probability proportional to the number of connections the member already has.

In our study we use two BA types of SF networks. One is the *hierarchical* network with directed connections [14,15], which is a history-dependent network in the sense that a member only listens to and can be convinced by the four people who joined earlier and were selected by the member. The four people, who are higher in the hierarchy than the new member, do not listen to the new network member (that is, do not change their opinion as a result of talking to the new network member). This is presumably the way a group with a rigid hierarchical command structure operates. Thus, one has a hierarchy determined by who joins the group first. The second type of the network that we use is *symmetrical* in the sense that all the connected members may influence each other, which is the way a group with a flexible command structure may operate, so that even if the top leaders (the original fanatics) are eliminated, the group and its influence on people's opinion may live on.

To simulate our model on a SF network, and to do so in a way that corresponds to the continuum model of Castillo-Chavez and Song [2], we adopt the following rules:

(see rules on top of page)

In this paper we mostly use, $\beta_i = \gamma_i$ ($i = 1, 2, 3$), and refer to their common value as p , as the main goal in this paper is to study the effect of a few well-connected fanatics on the opinion of an entire population. Since the behavior of the population now depends on the individuals' opinion and not just on their sum over all the lattice sites, sequential updating was used to simulate the model in both types of the network. We start with four fanatics on the network core while everybody else belongs to the general population G . We assume that the initial four fanatics are charismatic leaders forming the initial core of the network and, thus, becoming well-connected later.

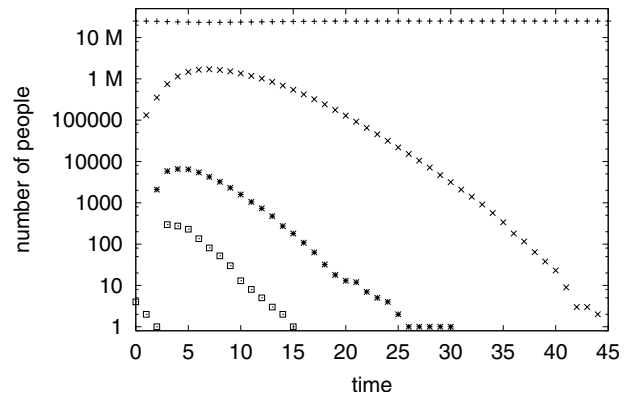


Fig. 1. Time development, from top to bottom, of the general (+), the susceptible (\times), the excited (*), and the fanatic population (squares) in the hierarchical network. The total population is 25 million; the vertical scale is logarithmic.

3 Results and discussion

Figure 1 shows the results using the hierarchical network. Here, we used the probability $p = 1/2$. It indicates that in the first few time steps few fanatics can convert more than a million people to being susceptible to their opinion in a population of 25 million, even though the number of the fanatics actually falls down in the first few steps. The E and F groups grow to much smaller percentages. Finally, the three groups, S , E , and F vanish, and everybody returns to the general population G . However, the S and E groups can survive longer than the fanatics; it is even possible that the fanatics die out accidentally after three time steps. Nevertheless, the avalanche that they set in motion stays on for a long time, which is in fact a well-known phenomenon for many extreme opinions or groups with such opinions.

In the symmetric, instead of hierarchical, networks the survival of the S and E groups is also possible, instead of their eventual extinction that Figure 1 indicates. This is shown in Figure 2. For a probability $p = 1/2$ to return from the S , E , and F groups to the general population G , the fanatics decrease from 4 to 2 in the first time step and vanish afterwards; nobody becomes excited, but up to 100 people become susceptible for some time, which is indicated by the continuous curve in Figure 2. If, however, we reduce to 0.1 (from $1/2$) the probabilities γ_i to return from the S , E , and F groups to G , then all four populations survive as large fractions of the population (shown by symbols in Fig. 2). The mutual reinforcement of opinions in symmetric networks, which is impossible in the hierarchical networks, greatly increases the spread of

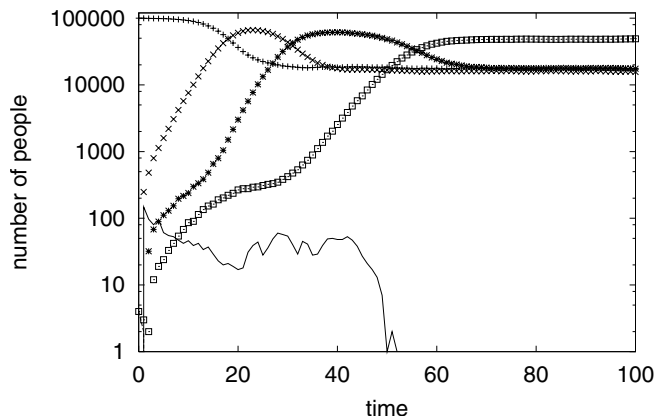


Fig. 2. Results with the symmetric networks with 10^5 people. With the usual parameters (see the text) the susceptibles first grow in numbers and then die out (line). With a 5 times slower rate of return to the general opinion, the G (+), S (\times), E (*) and F (squares) groups all become roughly equal and do not die out. After 100 iterations, the results remain stable at least up to 100 000 iterations.

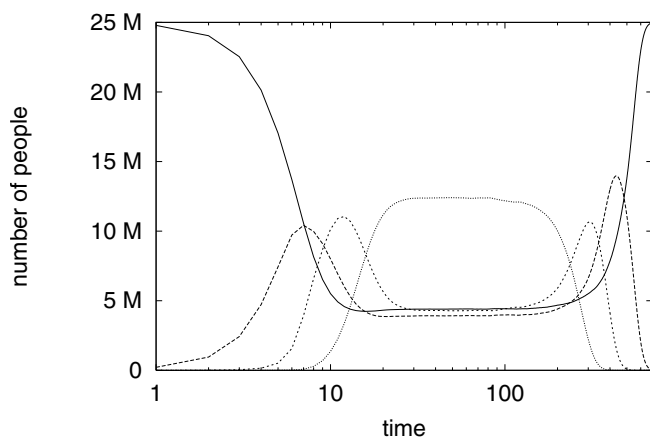


Fig. 3. Reduced probabilities as in Figure 2, but for a hierarchical network of 25 million people, with linear vertical and logarithmic horizontal axis. Eventually, everyone returns to the general population G as in Figure 1.

opinions. For comparison, Figure 3 shows the results for the hierarchical network with the same reduced probability $p = 0.1$ to return, indicating that even with this probability everybody becomes normal (returns to the general population) after some time, i.e., stops believing in the fanatics' opinion.

The great influence of the four initial fanatics comes from the fact that these founders of the network, numbers 1, 2, 3, and 4 in its history, are well connected. The later a person joins the interaction network (higher membership numbers), the smaller is, in general, the number of connections and, thus, the influence. This effect is shown in Figure 4 where the top curve shows how up to 5% of the population become susceptible under the influence of numbers 1, 2, 3, and 4 (taking $p = 1/2$). If, instead, network members 11, 12, 13, and 14 are taken as the initial radicals (second curve from above in Fig. 4), then less than 1%

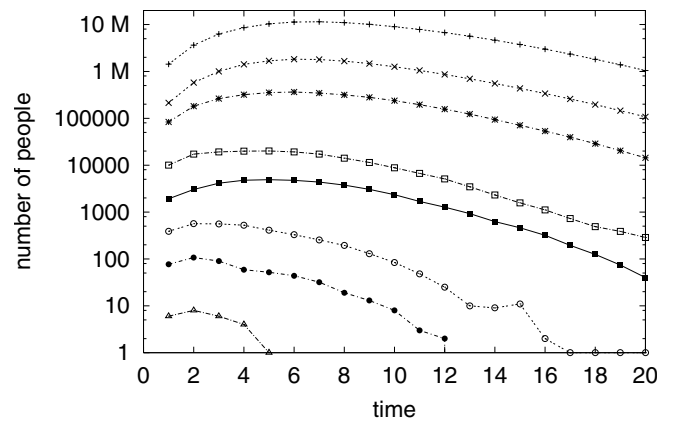


Fig. 4. The sum over all susceptibles in 10 hierarchical networks of 25 million people each. The four initial radicals joined the network, from top to bottom, as numbers 1, 2, 3, and 4; 11, 12, 13, and 14; then 101, 102, 103, and 104, until 10 000 001, 10 000 002, 10 000 003, 10 000 004 for the lowest curve. Late-comers are seen to have little influence.

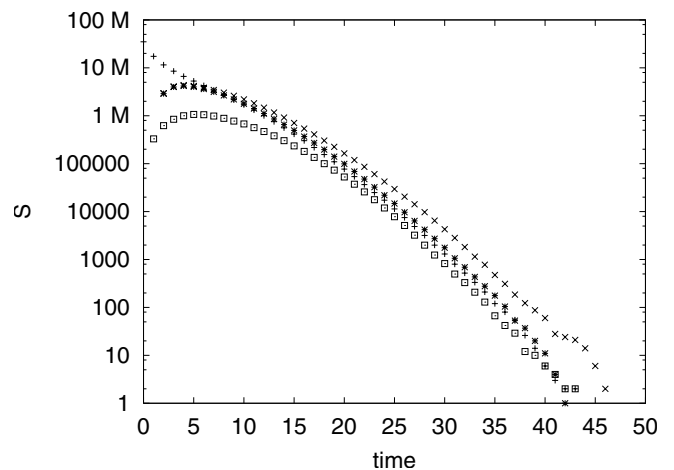
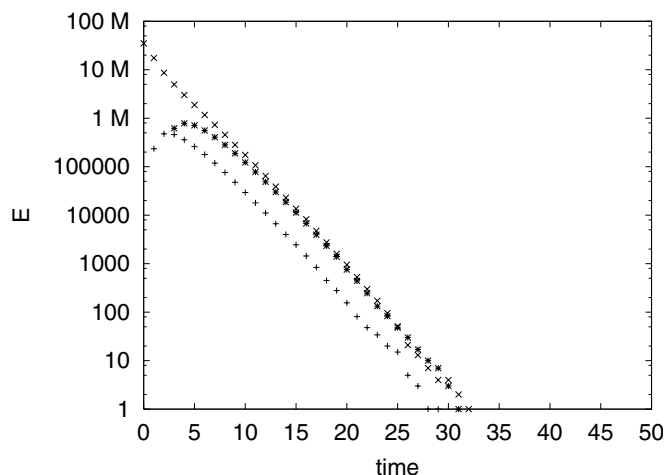
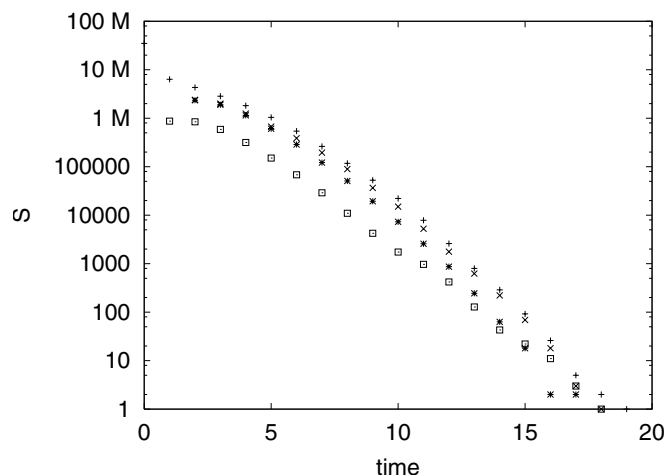
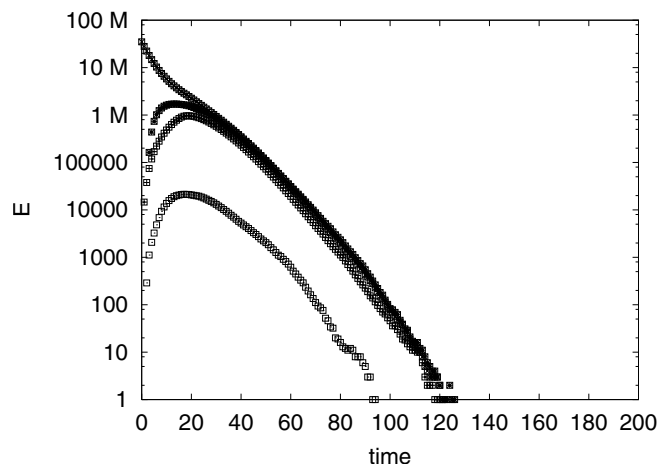
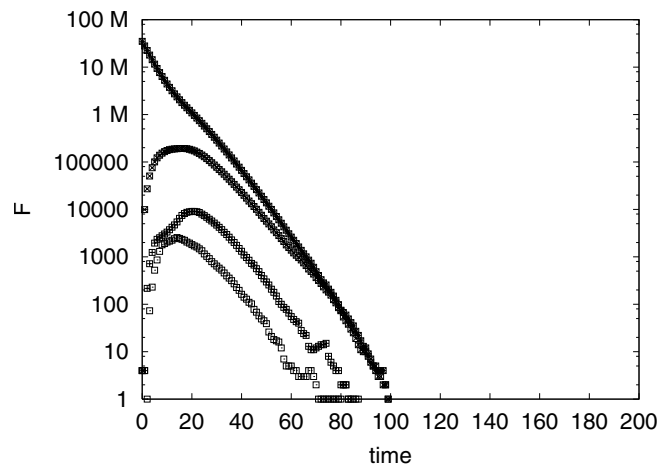


Fig. 5. The evolution of the susceptible population S when, aside from the core four fanatics, the rest of the population is initially susceptible (+), exited (\times), fanatic (*), or belongs to the general population (squares). The results are for the hierarchical structure obtained with the probability $p = 1/2$.

of the population becomes susceptible. The lower curves show analogously how the influence of the initial four fanatics is reduced if we take them as the four which follow numbers 10^2 , 10^3 , ..., 10^7 in the networks of size 25 million.

Due to the nonlinearity of the model, the initial concentrations, $E(0)$, $S(0)$, and $G(0)$, are important to its dynamics and, therefore, we have considered their effect. We considered the case in which everybody outside the initial core was initially, (a) susceptible (S); (b) excited (E); (c) fanatic (F), or (d) belonged to the general population (G) as before. The four core members were always the fanatics (F). We studied the model in the hierarchical SF network with 35 million nodes.

Figure 5 shows the results for the evolution of susceptible population S in the four cases, with the probability

Fig. 6. Same as in Figure 5, but for the excited population E .Fig. 8. Same as in Figure 5, but with $p = 0.8$.Fig. 7. Same as in Figure 6, but with $p = 0.2$.Fig. 9. Same as in Figure 6, but for the fanatic population F .

p being $1/2$. Except when the whole system (aside from the core four fanatics) is composed of susceptible people, the fraction of the S population first increases, reaching a maximum, but then decreases essentially exponentially, even when everybody in the network is initially a fanatic. A similar phenomenon is seen to happen to the excited population E , the results for which are shown in Figure 6. Such a behavior will not change if the probability p is varied. For example, Figure 7 presents the results for the E population obtained with $p = 0.2$, while Figure 8 shows those for the S population with $p = 0.8$. In all cases, the excited and susceptible populations eventually vanish. Even the population of the fanatics eventually vanishes with the hierarchical structure. For example, Figure 9 shows the results for the fanatic population with $p = 0.2$. The only effect that the probability of conversion has is the time scale over which the populations of the excited, fanatic, or susceptible people eventually vanish. Therefore, with a hierarchical structure everybody will eventually go back to the general population, and will neither be suscep-

tible to nor excited about the opinion originally advocated by the core fanatics. The most important aspect of these results is the robust nature of the model: regardless of the initial composition of the network, the E , F , and S segments of the population eventually die out, and everybody returns to the general population.

To see whether the connectivity and hierarchy of the network make any difference to the results shown in Figures 5–9, we repeated the simulations using the symmetrical SF network in which the influence of two connected nodes on each other is mutual. Figure 10 presents the results for the susceptible population, which should be compared with those shown in Figure 5. Similar to Figure 5, the susceptible population in this case also decreases over time, but the reduction, rather than being exponential as in Figure 5, is complex and seemingly resembles an oscillatory pattern, which is due to the feedback mechanism which is present in the symmetrical network.

BA networks have a percolation threshold [10] vanishing as $1/\log(N)$ and thus purely geometrically

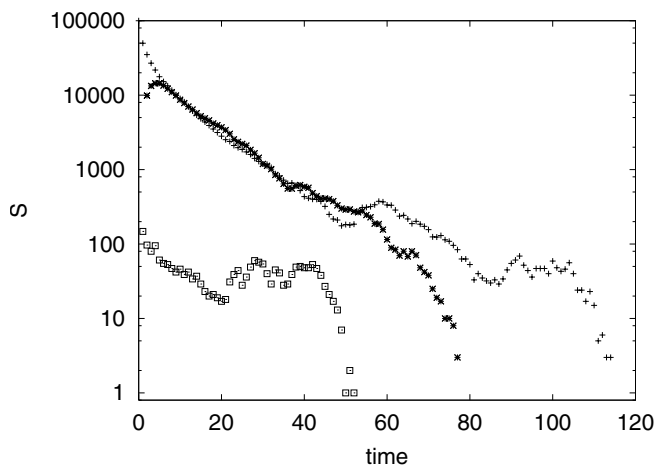


Fig. 10. Same as in Figure 5, but for the symmetrical SF network.

information can always spread through a large population. But this is only a necessary and not a sufficient condition for opinion spreading; as Figure 1 and the lower curve in Figure 2 indicate, opinions may also die out instead of spreading.

4 Summary

Although some previous works [16] had investigated the spreading of a state shared by a number of agents, none was in the context of the type of model that we study in this paper, namely, a four-component interacting system with the interactions being via a SF network. In addition, we find important differences between the influence of the hierarchical and symmetric networks on opinion dynamics. If the followers listen to the leaders but not the other way around (hierarchical interaction network), then the ideas of the leaders will die out. If, on the other hand, the leaders also listen to their followers, then their opinions spread only to a smaller fraction of the population but may last long, even if the leaders themselves are eliminated. The closer the leaders are to the core of the network (the best connected part of the network), the higher is their impact on the general population. This is similar to Ising magnets which were studied on SF networks [17] and square lattices and gave drastically different results for hierarchical as opposed to undirected symmetric interactions. In contrast, no such differences were observed in another model of opinion dynamics [15] networks. We regard the possibility of a few people to influence a large fraction of the population, and the persistence of an opinion in a symmetrical SF network but not in a hierarchical one, as the main results of this paper.

Further predictions of the model, a comparison with its continuum counterpart, and its simulation on regular two-dimensional lattices, is reported elsewhere [18].

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